Computational Requirements for Earth Gravity Field Determination

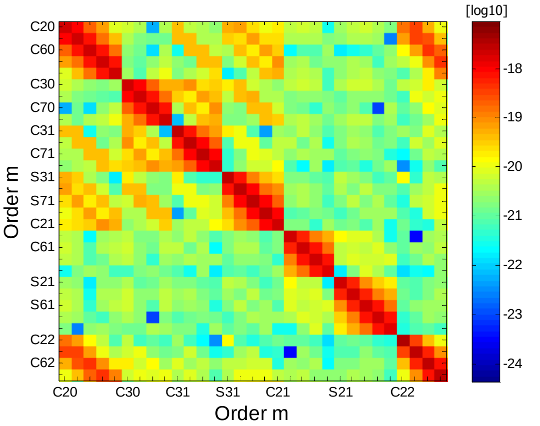
T. Fecher , T. Gruber, L. Zenner

Institut für Astronomische und Physikalische Geodäsie, Technische Universität München, Germany,

e-mail: fecher@bv.tum.de, thomas.gruber@bv.tum.de, zenner@bv.tum.de

Introduction

All globally given functions on the earth can be described in form of spherical harmonics. Hence every globally distributed quantity (e.g. geoid heights, gravity gradients, pressure etc.) is connected to a set of so-called potential coefficients (C_{nm}/S_{nm}). Several approaches for the determination of these coefficients exist, e.g. least squares adjustment. Least Square Methods require the solution of huge equation systems. This calls for immense computational and memory requirements and points out that the earth gravity field determination is a computational challenge. This poster shall give insight into some theoretical and computational aspects concerning earth gravity field determination.



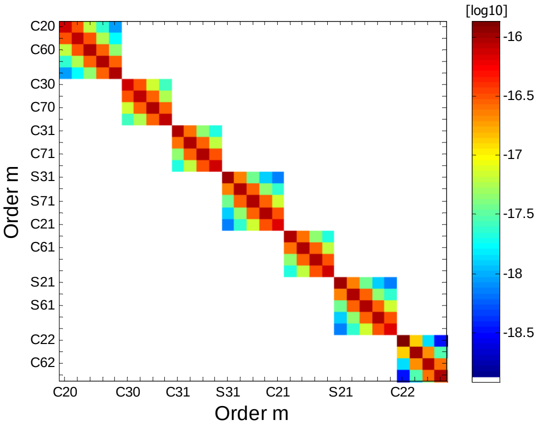


Fig. 1 (top) and Fig. 2 (bottom): Full and block-diagonal variance-covariance-matrix $(A^TPA)^{-1}$ of unknowns (sorted after order m and even/odd degrees).

Degree of development	Number of unknowns	Normal equation elements [Mill.]	Size of full equation system [GB]	Size of block diagonal system [GB]
180	32761	1,073	8	0.008
360	130321	16,983	127	0.06
720	519841	270,234	2013	0.5
2190	4800481	23,044,617	171696	13

Tab. 1: Number of unknowns and elements of the normal equation matrix.

Computation of the Earth Gravity Field

The basic formulas for the least squares adjustment are:

$$\hat{x} = (A^TPA)^{-1}A^TPl \quad \& \quad Q_{xx} = \sigma^2(A^TPA)^{-1}$$

Unknowns are the potential coefficients C_{nm}/S_{nm}.
The observation **l** is a global function usually given in discrete measurement points and can be interpolated on a regular global (θ,λ)- grid.

Example for observation equations l are:

Geoid heights:

$$N = R\sum_{n=2}^{\infty}\left(\frac{R}{r}\right)^{n+1}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\cdot\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]$$

Radial gravity tensor component:

$$T_{zz} = \frac{GM}{R^3}\sum_{n=2}^{\infty}(n+1)(n+2)\left(\frac{R}{r}\right)^{n+3}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\cdot\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]$$

Atmospheric pressure

$$Pr_n = -\frac{Mg}{R^2}\frac{2n+1}{1+k_n}\sum_{m=0}^{n}\bar{P}_{nm}(\cos\theta)\cdot\left[\bar{C}_{nm}\cos m\lambda + \bar{S}_{nm}\sin m\lambda\right]$$

(R=radius of the sphere; r, θ, λ=spherical geocentric coordinates; $\bar{P}_{nm}$ =fully normalized associated legendre polynomials;

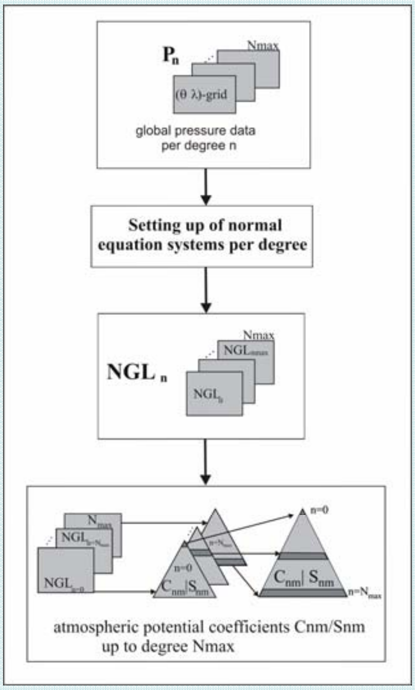
$\bar{C}_{nm}$, $\bar{S}_{nm}$ =fully normalized potential coefficients; G=gravitational constant ; M=earth mass,

k_n =loading love numbers; n/m=spherical harmonic degree/order)

The partial derivatives of **l** with respect to the unknown coefficients C_{nm} and S_{nm} are required for the setup of the coefficient-matrix **A**. The weighting-matrix of the observations **P** is given by the uncertainties of **l**. The normal equation system (A^TPA) can then be set up and the unknown potential coefficients C_{nm}/S_{nm} can be determined.

The most computational effort is needed for the setup and inversion of the normal equation system (A^TPA). In case of equally weighted observations on a regular grid, the full normal equation system (Fig. 1) is reduced to a block-diagonal system (Fig. 2) and can be solved by less computational effort (see Tab. 1).

Atmosphere processing (6 hourly)



Atmosphere processing. To determine the atmospheric potential coefficients C_{nm}/S_{nm} the pressure at the centre of mass of the atmospheric column has to be taken into account. That means full normal equation systems per spherical harmonic degree have to be set up. Consequently the computational requirements expand with increasing degree n (see Tab. 2).

Degree of development	Number of unknowns	Number of normal equations to solve	Size of one equation system	Size of all equation systems
10	121	1364	72 KB	100 MB
30	961	3844	4 MB	14 GB
50	2601	6324	28 MB	170 GB

Tab. 2: Computational requirements for atmosphere processing (one month with a resolution of 6 hours).