Earth Gravity Field Determination from Space – A Computational Challenge

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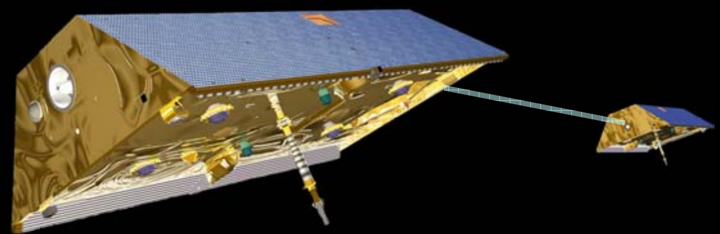
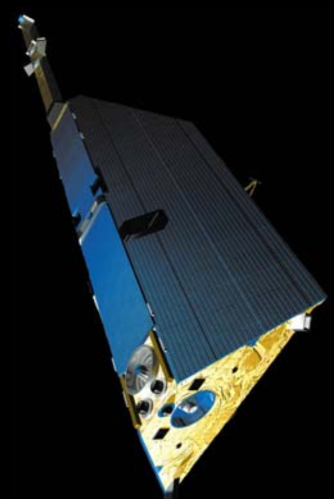
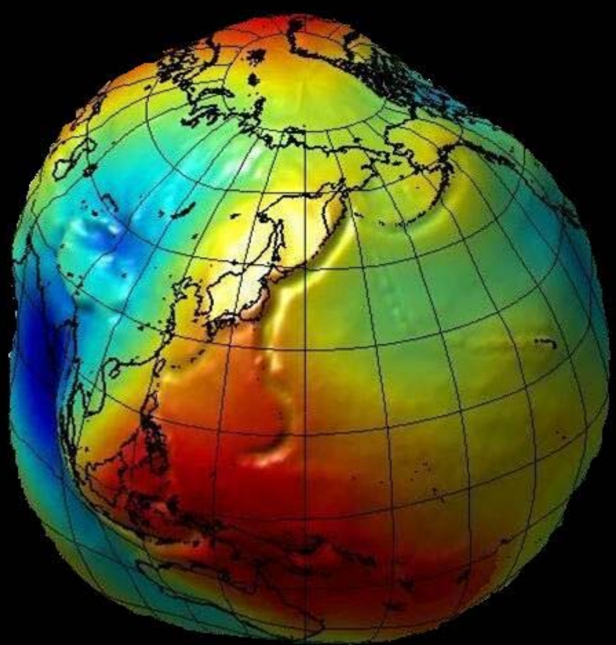
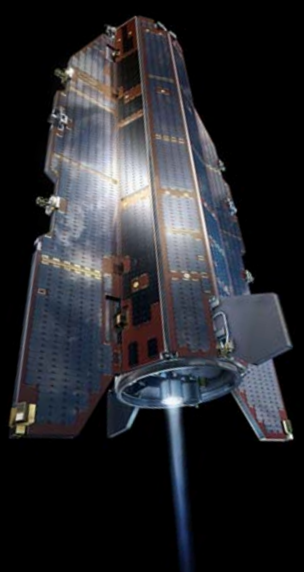
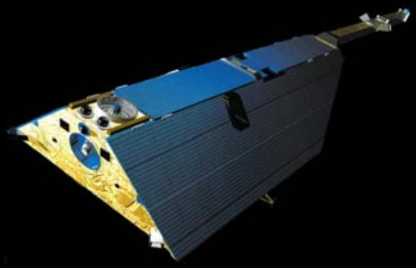




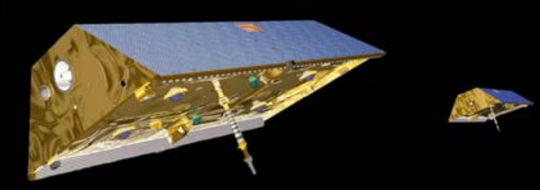




LRZ LINUX Cluster Workshop, 8.12.2008



CHAMP



GRACE

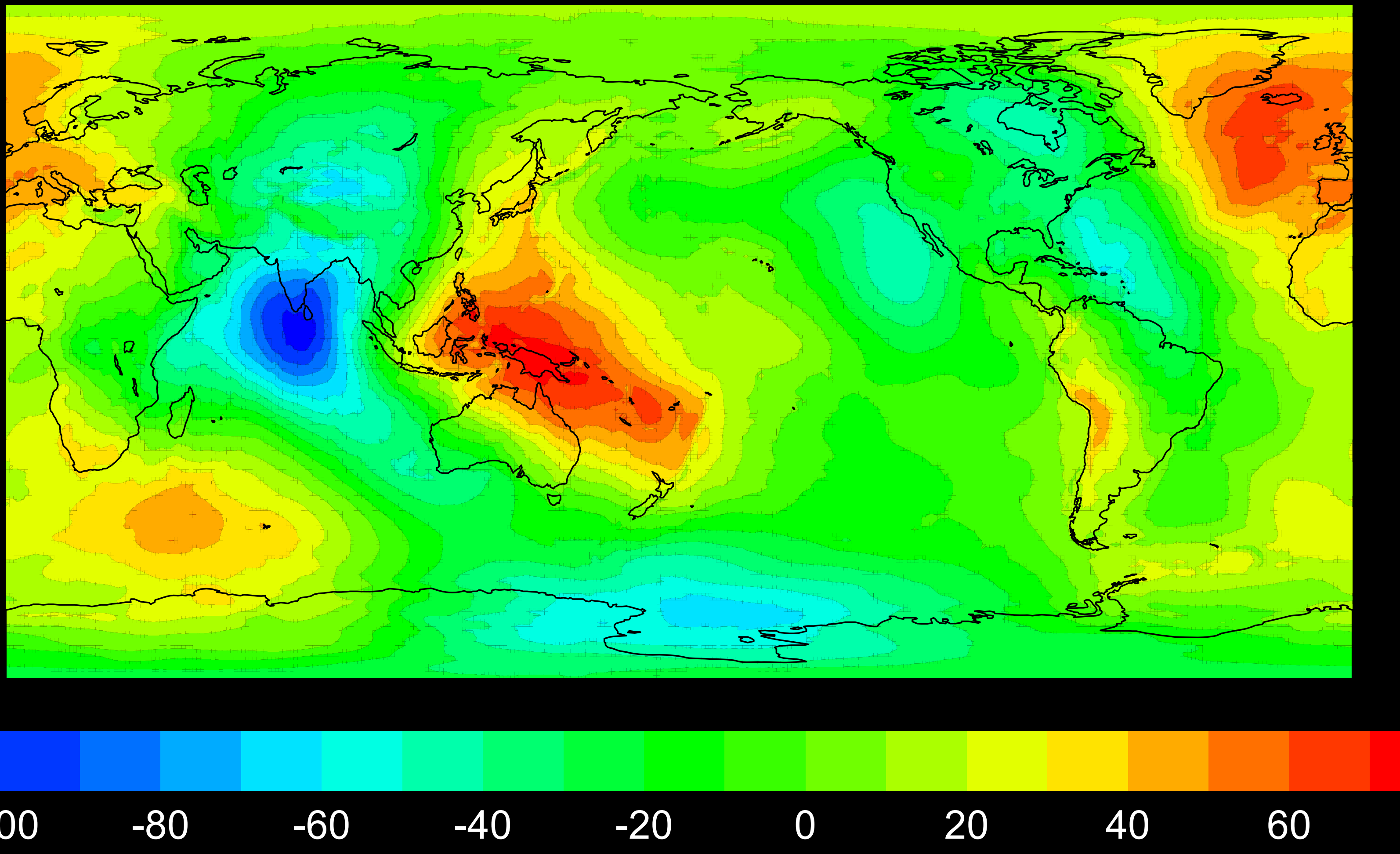
GOCE

Outline

1. The Earth Gravity Field:
   A few Basics
2. How do we Observe
   Gravity from Space?
3. Computational Challenges
4. Summary & Conclusions

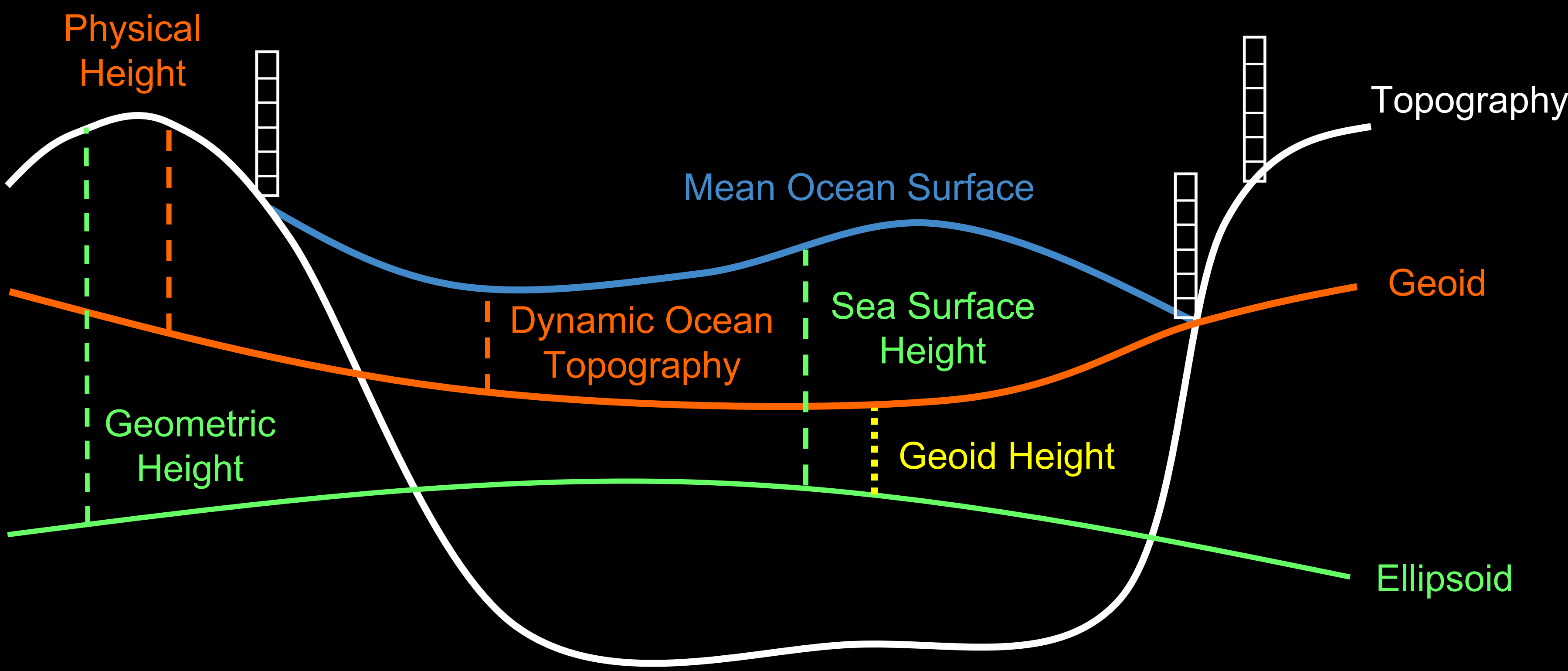
The Earth Gravity Field

The Geoid



The Earth Gravity Field

The Geoid



Equipotential Surface = Surface of equal Gravity

Reference of Physical Height Systems

The Earth Gravity Field

Mathematical Model

Disturbing Potential

$$T = \frac{GM}{r} \sum_{n=0}^{N} \left(\frac{a}{r} \right)^n \sum_{m=0}^{n} \left(C_{nm} \cos m\lambda + S_{nm} \sin m\lambda \right) P_{nm} \left(\cos \theta \right)$$

Geoid Height

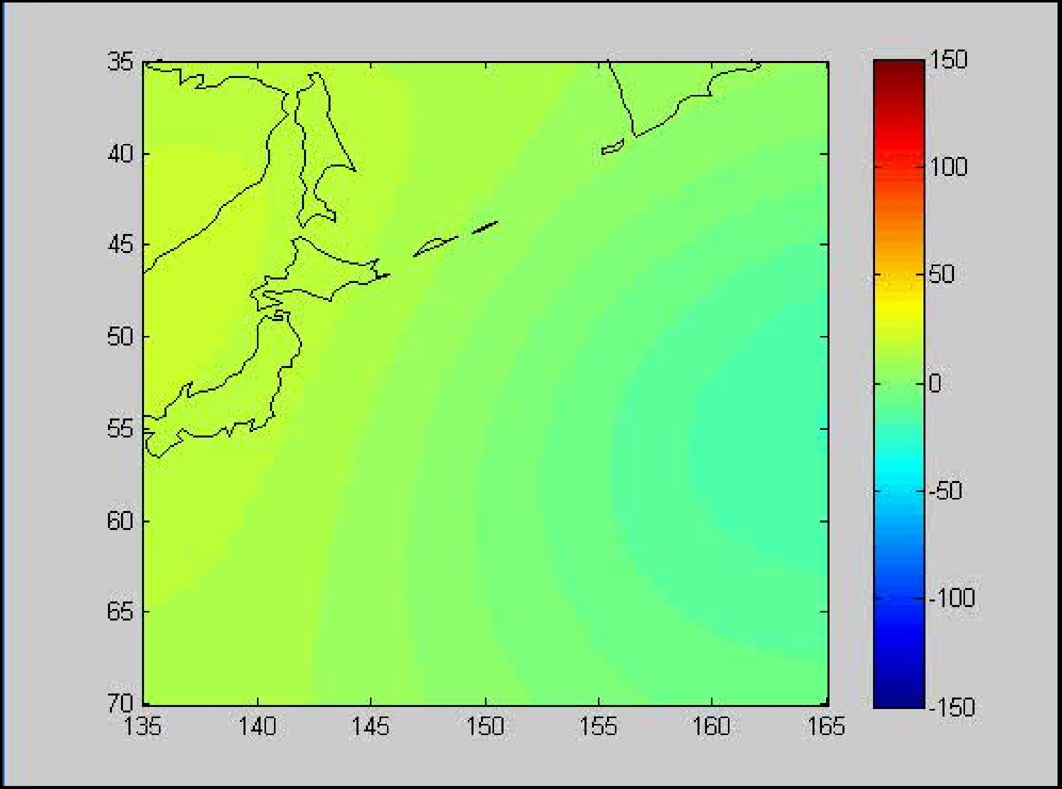
$$N = \frac{T}{\gamma}$$

Gravity Anomaly

$$\Delta g = -\frac{\partial T}{\partial r} - \frac{2}{r} T$$

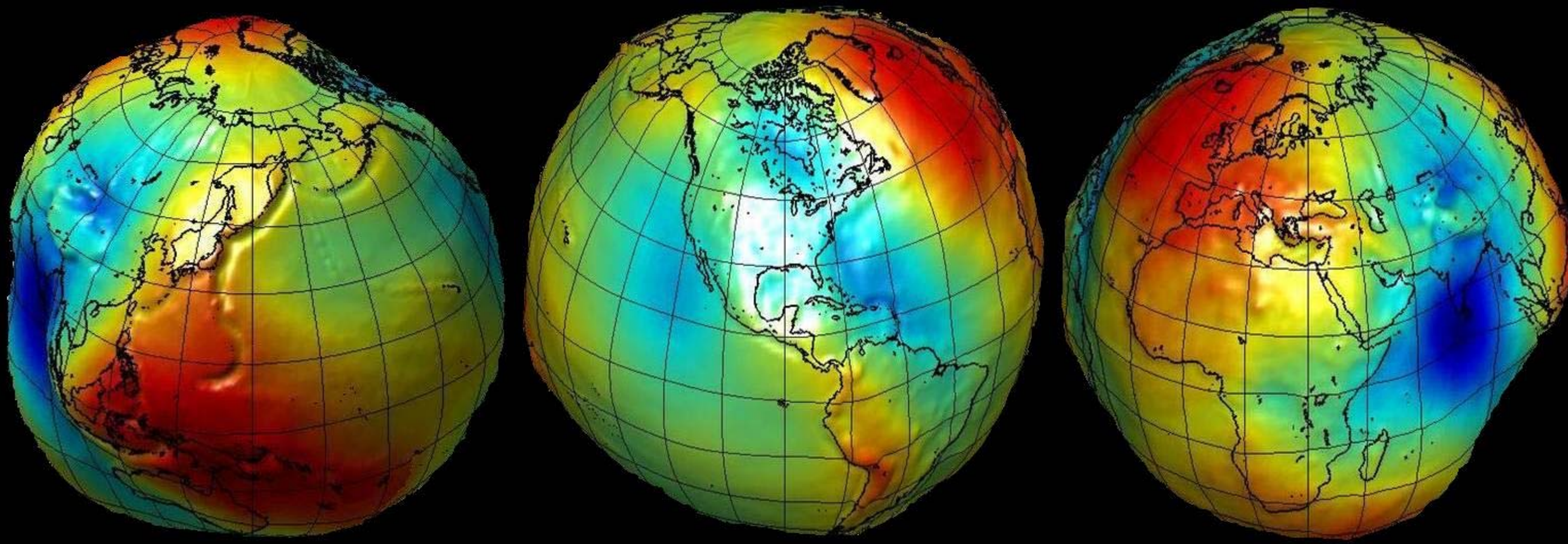
Gravity Gradient

$$T_{rr} = \frac{\partial^2 T}{\partial^2 r}$$



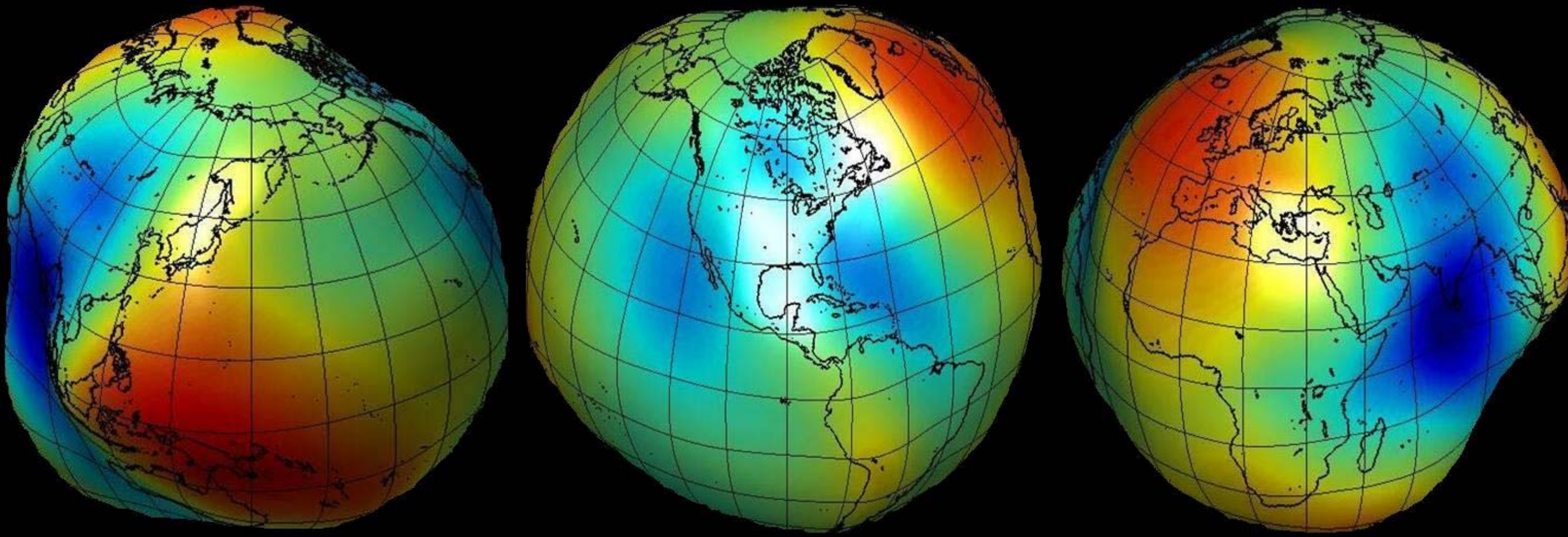
The Earth Gravity Field

Seen on Ground



The Earth Gravity Field

Seen in 500 km Height



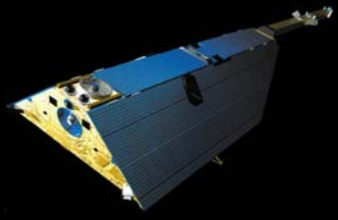
Observing Gravity Field from Space

Boundary Conditions of Mission Design

1. A satellite can be regarded has a free-falling test mass in the Earth gravity field.
2. Due to quadratic attenuation of gravity signal with height the satellite has to fly as low as possible.
3. Low orbits imply higher impact of non-gravitational forces, which have to be modeled or observed and/or compensated.
4. The orbit of a satellite (positions and velocities) represents an indirect gravity field observation. Therefore continuous positioning of satellite is required.
5. Signal attenuation partly can be compensated by observing derivatives of the gravity potential in space.

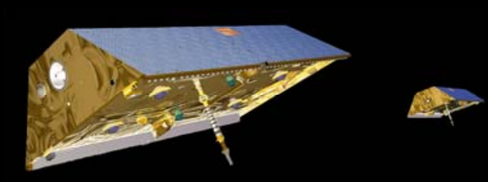
Observing Gravity Field from Space

Status of Satellite Missions



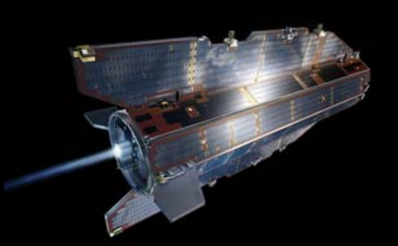
CHAMP

Launch: 15.7.2000
Funded by: GFZ Potsdam / DLR
Lifetime: 8 years (extension possible)
Orbit: 87 degree inclination, 450-300 km height
Goals: static gravity & magnetic field, atm. sounding



GRACE

Launch: 17.3.2002
Funded by: NASA / DLR, Science CSR Austin & GFZ
Lifetime: 8 years (extension possible)
Orbit: 89 degree inclination, 450-300 km height
Goals: static & time variable gravity field



GOCE

Launch: planned for 1. quarter 2009
Funded by: ESA
Lifetime: 2 years (extension possible)
Orbit: 96.5 degree inclination, 260-270 km height
Goals: static gravity field

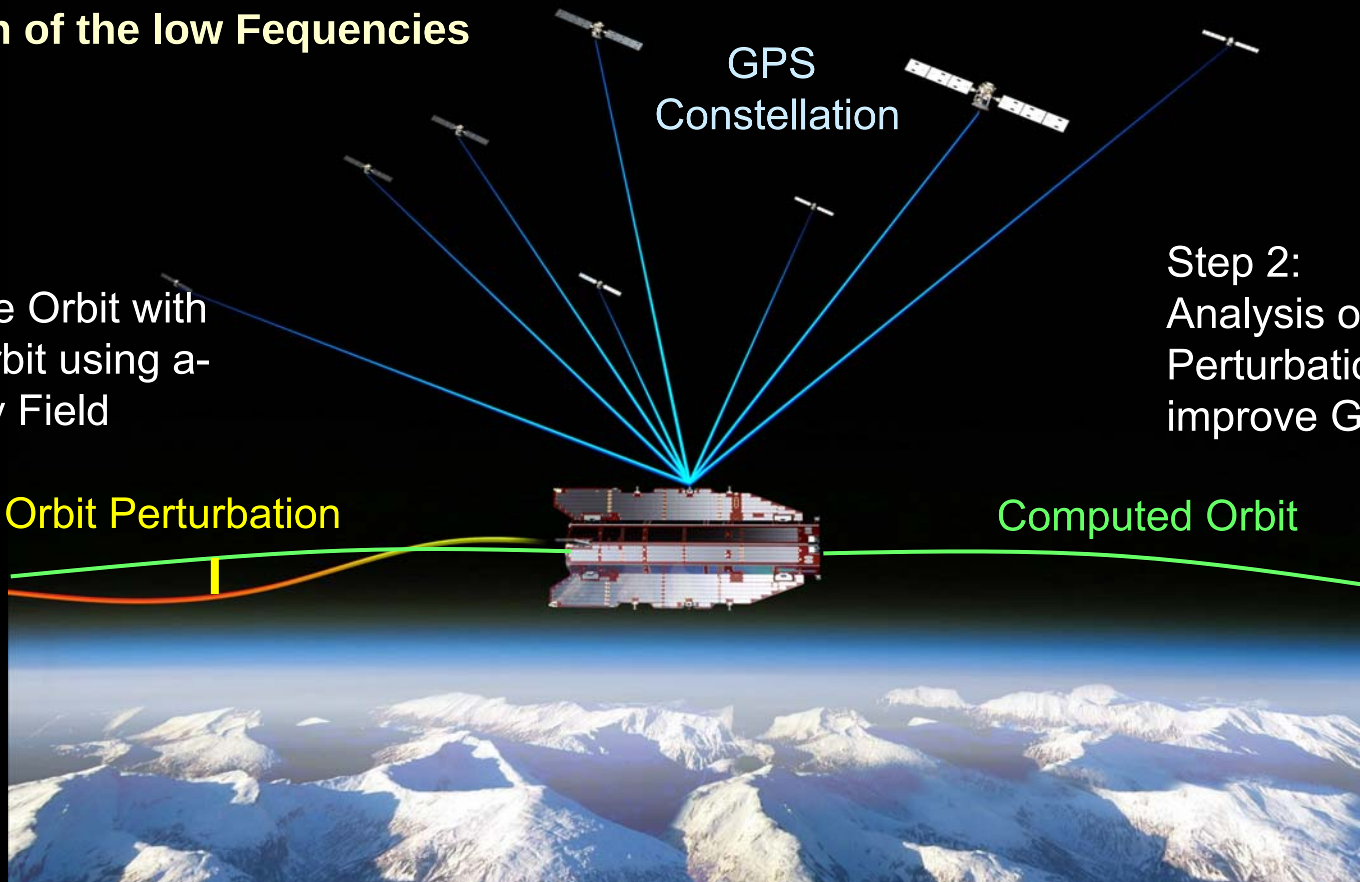
Observing Gravity Field from Space

Satellite-to-Satellite Tracking – High Low Case

Observation of the low Fequencies

Step 1:
Compare true Orbit with computed Orbit using a-prioiri Gravity Field

Step 2:
Analysis of Orbit Perturbations to improve Gravity Field

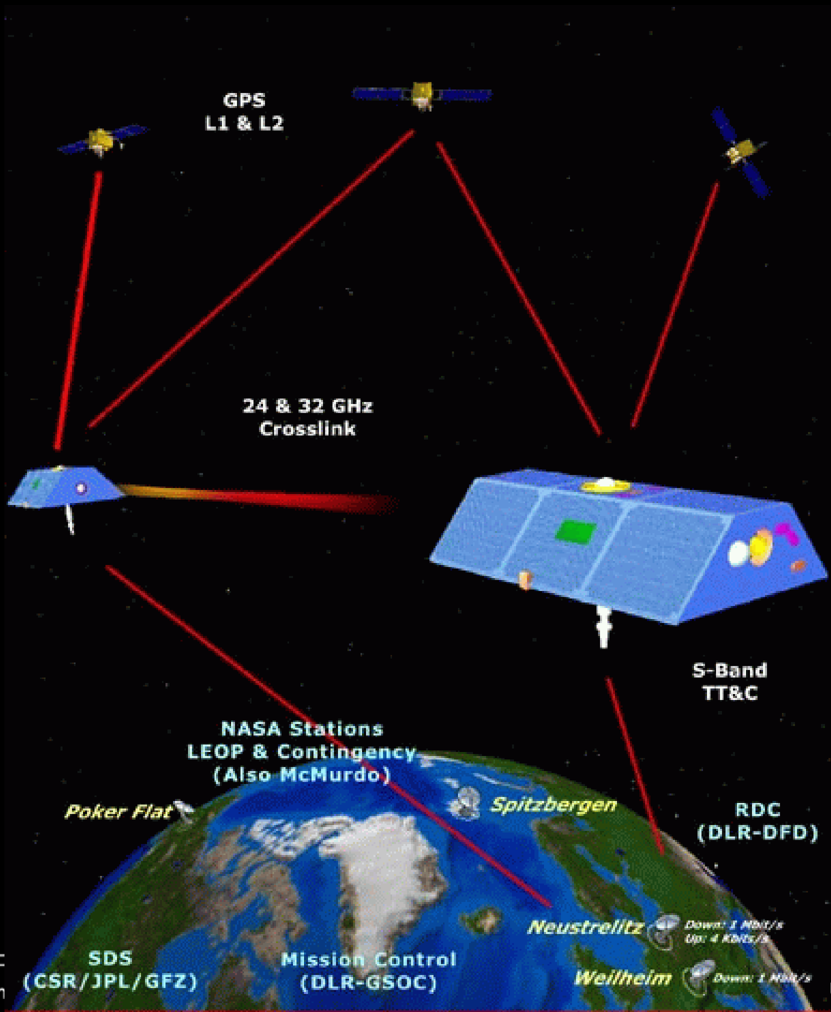


One Satellite is the Test Mass: e.g. CHAMP, GRACE or GOCE

Observing Gravity Field from Space

Satellite-to-Satellite Tracking – Low Low Case

Observation of the medium Fequencies by Analysis of Orbit Perturbations



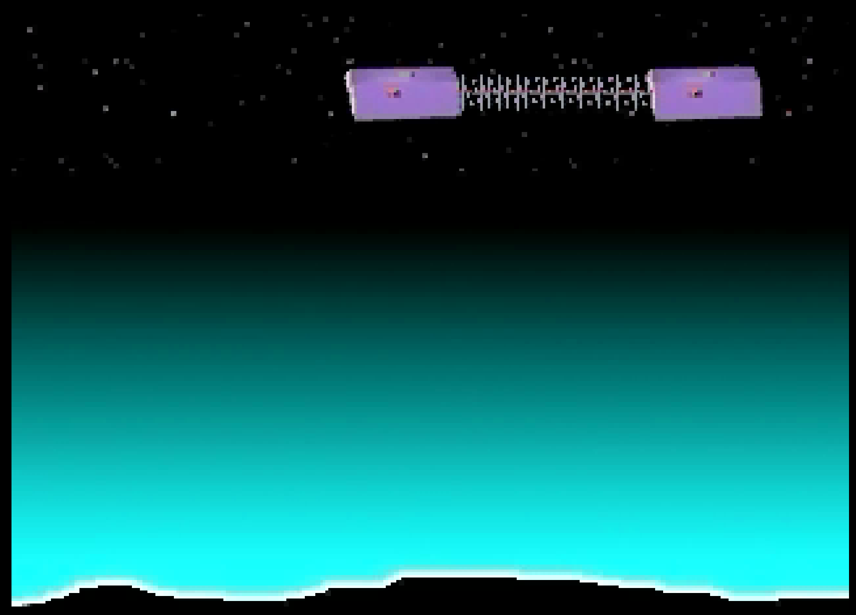


Illustration & Animation ©CSR

Two Satellites are the Test Masses observing each other: e.g. GRACE

Observing Gravity Field from Space

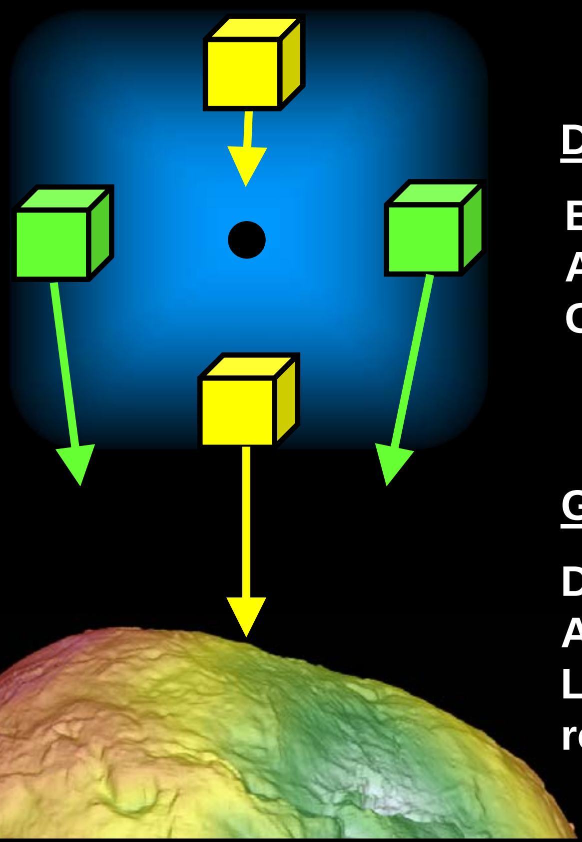
Gravity Gradiometry

Observation of the medium to high Fequencies

<u>Observation</u>

6 Accelerometers measure Accelerations in 3 Directions.

Measurement Accuracy: 10^{-12} ms^{-2}



<u>Differential Mode Accelerations</u>

By Subtraction of Accelerations along 1 Gradiometer Arm.

<u>Gravity Gradients</u>

Divide differential Accelerations by Arm Length and correct for rotational Accelerations

<u>Common-Mode Accelerations</u>

By Computation of Mean Value of Accelerations along 1 Gradiometer Arm.

The Satellite directly observes 2nd Derivatives of the Gravity Potential: e.g. GOCE

Computational Challenges

Observations & Parameters

Number of Observations for Satellite Instrumentation:

Space GPS Reveicer: 0.1 - 1 Hz, 12 channels

Accelerometer: 0.2 - 1 Hz

Intersatellite Microwave Link: 0.2 Hz

Gradiometer: 1 Hz

high-low SST: 1 Mio./day

low-low SST: 30.000/day

Gradiometry: 260.000/day

Surface Data: up to 100 Mio points

Number of Parameters depends of chosen maximum degree of SHS:

$$n_{Par} = (N + 1)^2$$

For high-low SST – CHAMP:	$N = 80$	$n_{Par} = 6.561$
For low-low SST – GRACE:	$N = 180$	$n_{Par} = 32.761$
For Gradiometry – GOCE:	$N = 250$	$n_{Par} = 63.001$
For Combination – Sat. & Surface Data:	$N = 2160$	$n_{Par} = 4.669.921$

Computational Challenges

Integration of Equation of Motion

(needed for Orbit Perturbations Analysis)

$$\ddot{\boldsymbol{r}} = -\frac{GM}{R^3}\boldsymbol{r} + \boldsymbol{F}_S$$

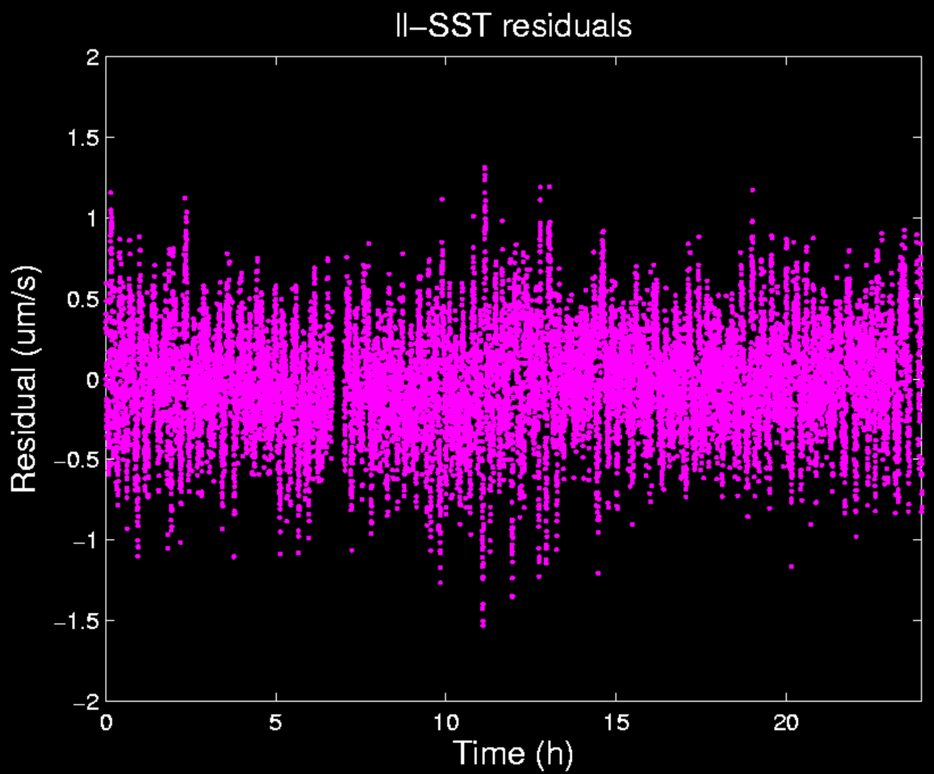
Numerical Integrator Requirements:

- for position observations [mm]: 11 digits
- for inter-satellite range obs. [0.1µm]: 15 digits

with:

$\boldsymbol{r}$	Position vector in inertial frame
$\ddot{\boldsymbol{r}}$	Acceleration vector in inertial frame
GM	Gravity constant times Earth mass
R	Distance from geocenter
$\boldsymbol{F}_S$	Disturbing force vector

- anomalous Earth gravity field
- direct and indirect tidal forces
- Non-gravitational (surface) forces: atm. drag, solar radiation, Albedo



Computational Challenges

Least Squares - Accumulation of Normal Equations

Least Squares Adjustment:

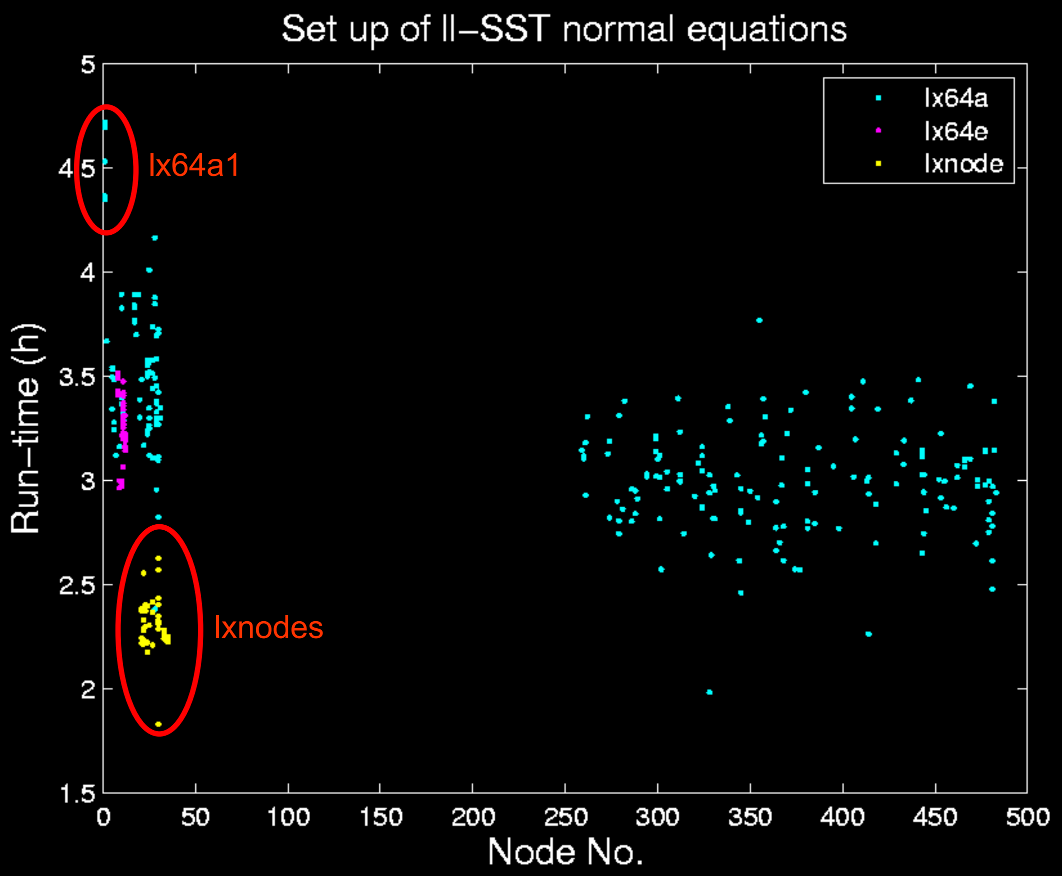
$$\boldsymbol{Ax} = \boldsymbol{l} + \boldsymbol{v}$$

$$E\{\boldsymbol{v}\} = 0 \qquad \boldsymbol{v}^T \boldsymbol{P}\boldsymbol{v} = min.$$

$$\boldsymbol{x} = \left(\boxed{\boldsymbol{A}^T \boldsymbol{P}\boldsymbol{A}}\right)^{-1} \boldsymbol{A}^T \boldsymbol{P}\boldsymbol{l}$$

$$\boldsymbol{Q}_{xx} = \sigma^2 \left(\boldsymbol{A}^T \boldsymbol{P}\boldsymbol{A}\right)^{-1}$$

$$\sigma^2 = \frac{\left(\boldsymbol{v}^T \boldsymbol{P}\boldsymbol{v}\right)}{n_{Obs.} - n_{Par}}$$



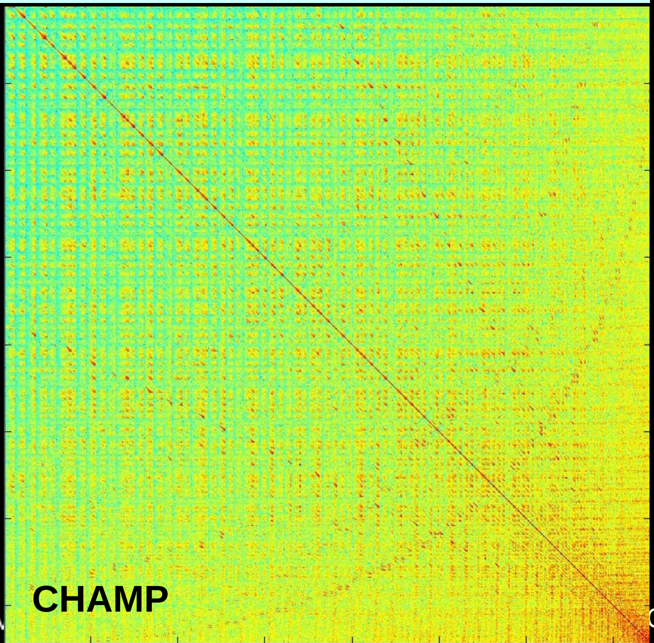
Computational Challenges

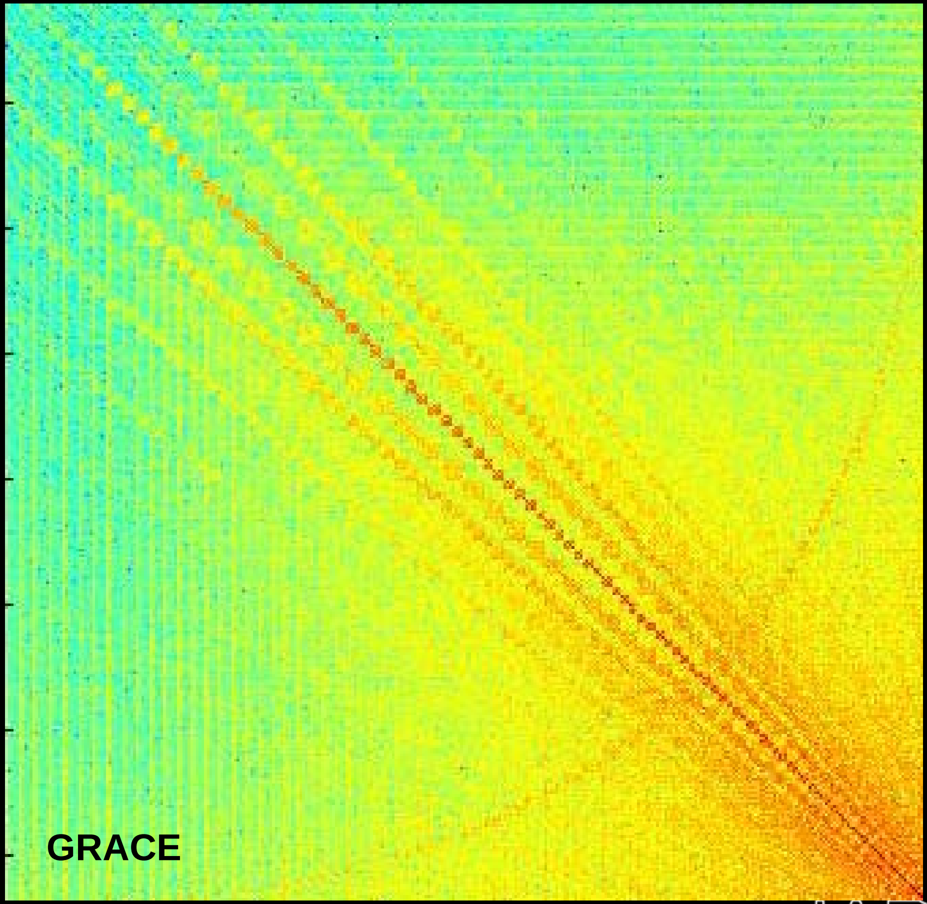
Least Squares – Solution/Structure of Normal Equations

Size of System to be inverted:

N	No. Matrix Elements	Matrix Size (8Byte)
80	43×10^6	350 MB
180	1×10^9	8.5 GB
250	4×10^9	32 GB
2160	22×10^{12}	175 TB

$$Q_{xx} = \sigma^2 \left(A^T P A \right)^{-1}$$

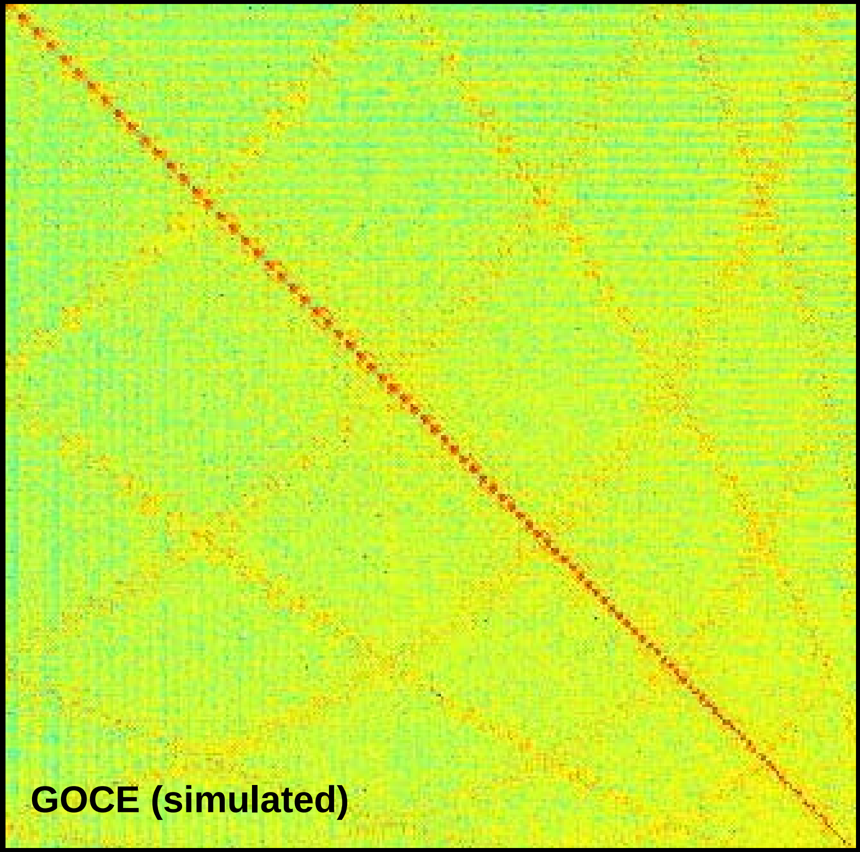


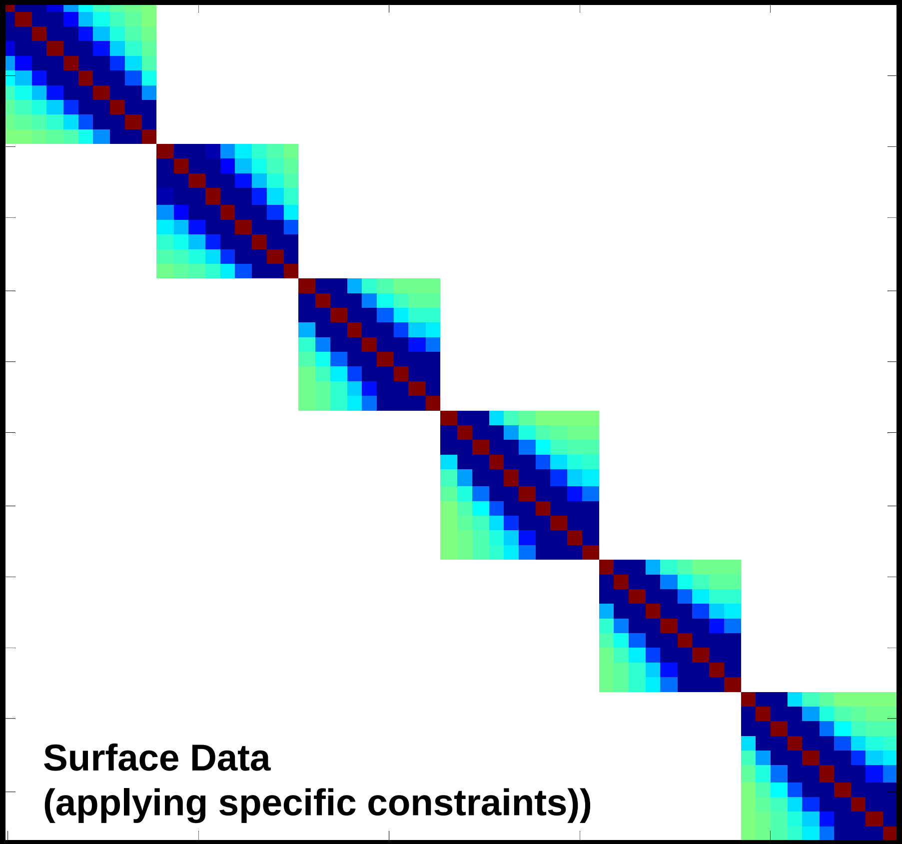


Computational Challenges

Least Squares – Solution/Structure of Normal Equations

$$Q_{xx} = \sigma^2 \left(A^T P A \right)^{-1}$$





Summary & Conclusions

- Satellites observing of the Earth Gravity Field from Space are providing a huge amount of data.
- The number of parameters to be determined strongly depends on the resolution (sensitivity) of the field to be determined.
- The global Earth gravity field is determined by an inverse process, which requires the setup and solution of large equation systems.
- For highest resolutions using surface observations simplifications have to be introduced in order to reduce the matrix size.
- The LINUX Cluster heavily is used for CHAMP and GRACE processing. It is the workhorse for our gravity field analysis.
- For GOCE we envisage additional computational demands. For this reason LRZ will host a „LINUX Geodesy Cluster“ currently under procurement.