



Challenges of a 1 cm Geoid from GOCE

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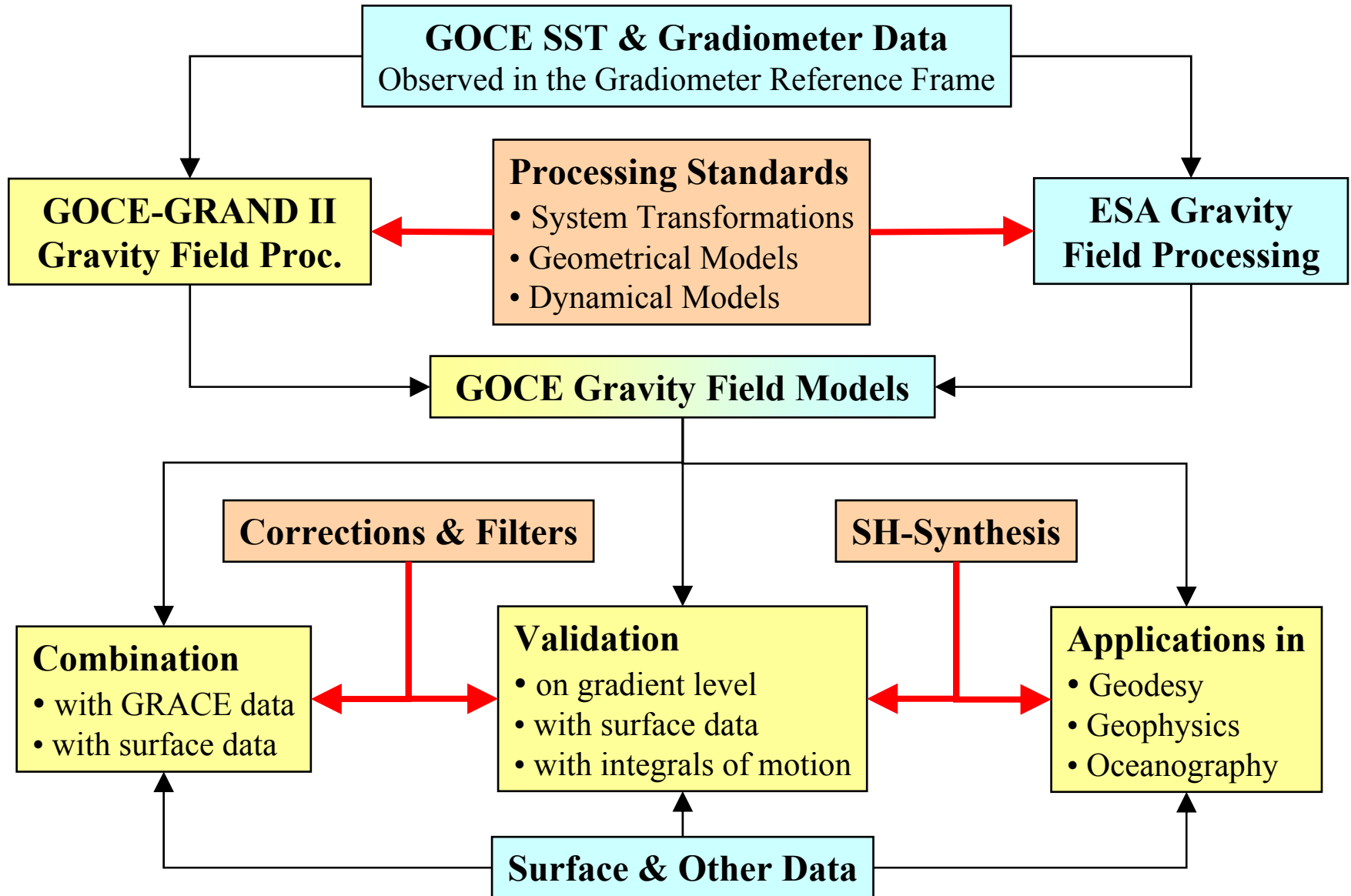
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Technische Universität München

GEOTECHNOLOGIEN Statusseminar, Beobachtung des Systems Erde aus dem Weltraum




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Challenges for 1 cm Geoid





GOCE-GRAND II Working Groups

Within GOCE-GRAND II three Working Groups have been established in order to work on common issues in a coordinated approach. These are:

GOCE Standards	System Transformations; Geometrical Models; Dynamical Models.	
GOCE and Surface Data	Combination & Validation; Corrections to be applied; Data Weighting; Aliasing & Filtering Techniques.	
GOCE and Applications	Identification of Needs; Zero Potential; Derived Gravity Field Quantities on Ellipsoid or Earth Surface.	


GOCE Standards

		<i>GOCE Standards</i> Doc. Nr: GO-TN-HPF-GS-0111 Issue: 1.2 Date: 16.01.2006 Page: 1 of 72
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GOCE High Level Processing Facility

GOCE Standards

Doc. No.: GO-TN-HPF-GS-0111
Issue: 1
Revision: 2
Date: 16 / 01 / 2006



Prepared by: The European GOCE Gravity Consortium
EGG-C

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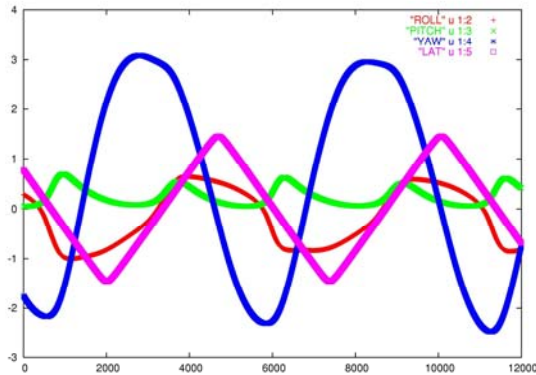
GOCE Standards implemented for ESA's High level Processing Facility:

- Numerical Standards
- Time Systems
- Reference Systems Definitions
- Transformation between Reference Systems
- Geometrical Models
- Dynamical Models
- Height, Gravity & Tide Systems
- GOCE Reference & Time Systems

GOCE Standards have been made available to GOCE-GRAND II Team !

GOCE Reference Frames

GRF vs. LORF



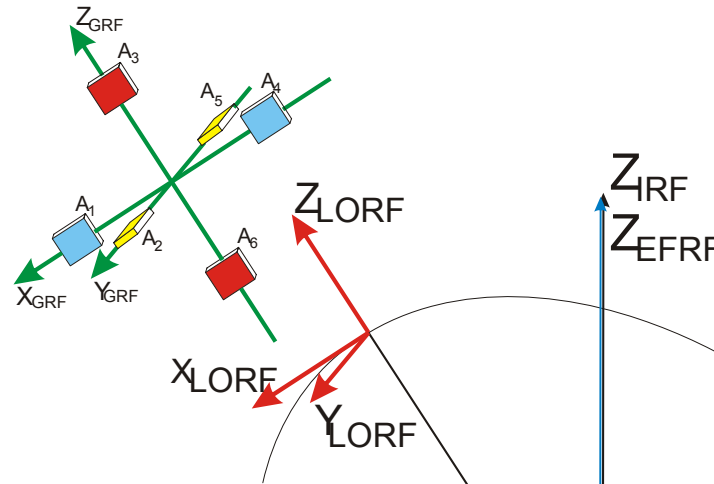
Courtesy: P. Visser / DEOS

GRF vs. IRF

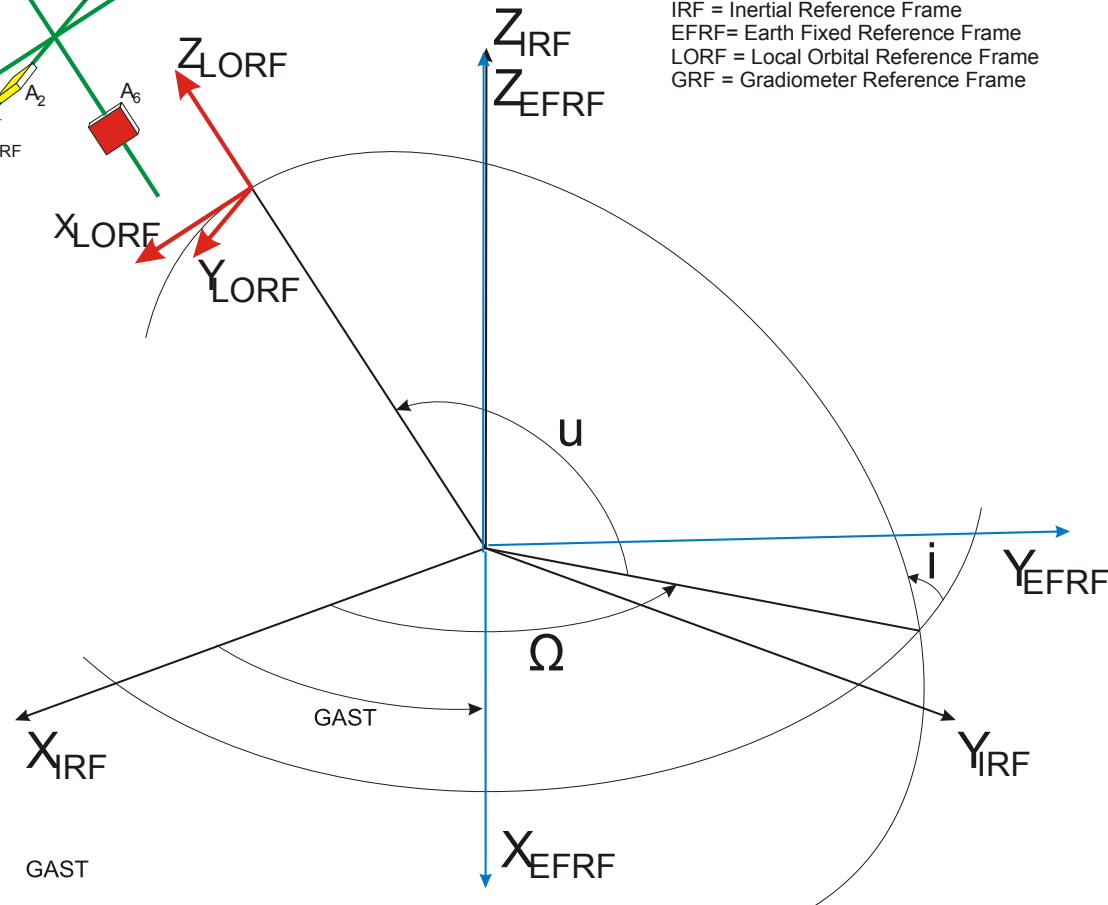
observed by 3 star cameras (with a few arcsec accuracy) and rotational information from the gradiometer.

EFRF vs. IRF

following IERS conventions 2003



IRF = Inertial Reference Frame
 EFRF = Earth Fixed Reference Frame
 LORF = Local Orbital Reference Frame
 GRF = Gradiometer Reference Frame

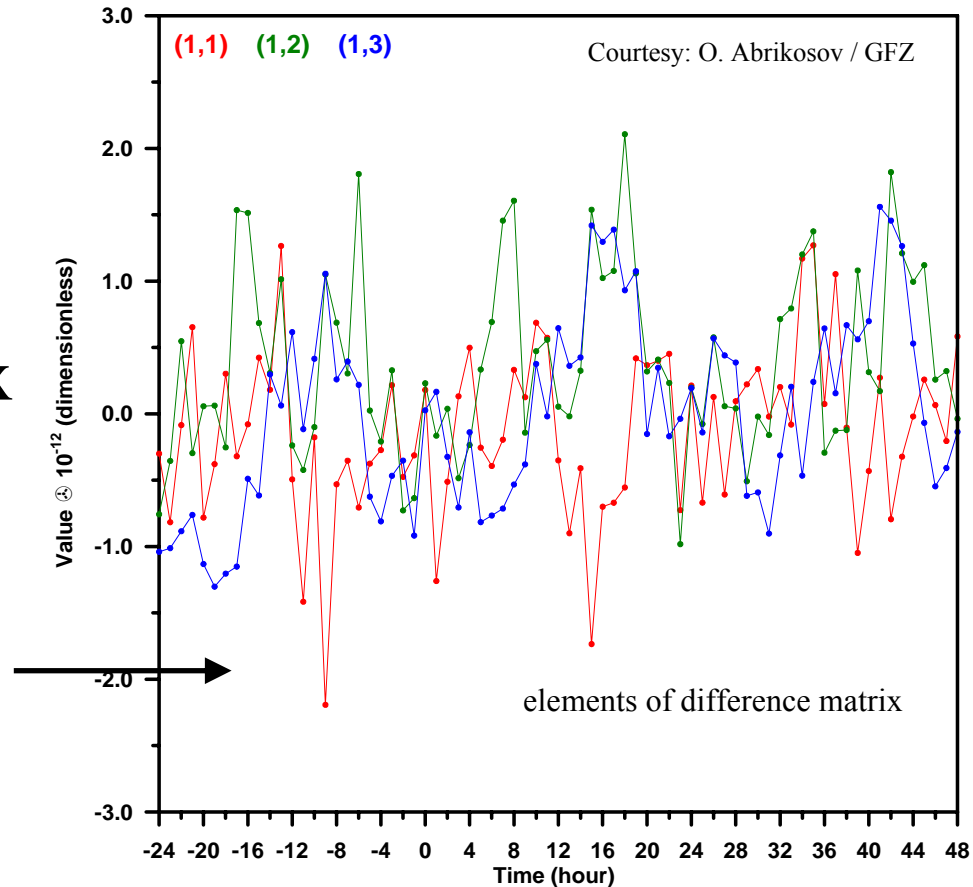


EFRF ↔ IRF

$$\mathbf{r}^{IRF} = \mathbf{B}(t)\mathbf{P}(t)\mathbf{N}(t)\mathbf{S}(t)\mathbf{W}(t)\mathbf{r}^{EFRF}$$

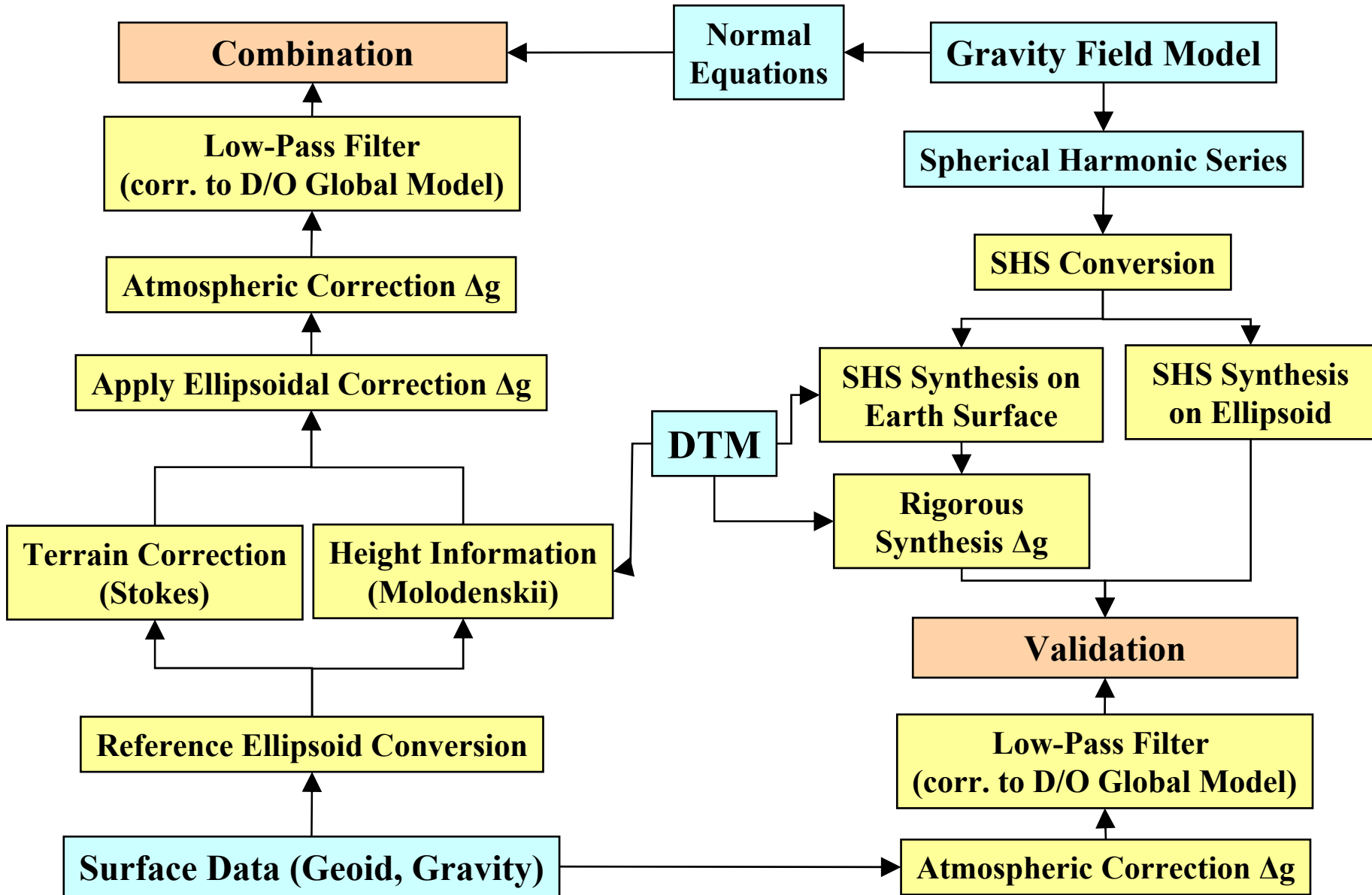
- B(t):** Frame bias matrix.
- P(t):** Precession matrix.
- N(t):** Nutation matrix
- S(t):** Earth rotation matrix
- W(t):** Polar motion matrix

Rotation matrix differences between IERS interactive tool and GFZ implementation.



Maximal deviations from the IERS results do not exceed 4.5×10^{-12} . This corresponds to the displacement of 0.03 mm in a point position at GOCE altitude, or to the uncertainty of 0.9 microarcseconds in the frame rotation.

GOCE & Surface Data



GOCE & Surface Data

Reference Ellipsoid Conversion Geoid vs. Spherical Harmonic Series Conversion

Given: $N^{(1)}, GM^{(1)}, a^{(1)}, f^{(1)}, \omega^{(1)}$

Given: $C_{nm}^{(2)}, S_{nm}^{(2)}, GM^{(2)}, a^{(2)}$

Generalized Brun's Equation
P: Geoid Point; Q: Ellipsoid Point

$$N = \frac{T_P}{\gamma_Q} - \frac{W_P - U_Q}{\gamma_Q}$$

Assumption: $W_P = U_Q$

$$\begin{aligned} N^{(2)} &= N^{(1)} + \frac{T_P^{(2)}}{\gamma_Q^{(2)}} - \frac{T_P^{(1)}}{\gamma_Q^{(1)}} = \\ &= N^{(1)} + \frac{W_P - U_P^{(2)}}{\gamma_Q^{(2)}} - \frac{W_P - U_P^{(1)}}{\gamma_Q^{(1)}} \end{aligned}$$

Approximations:

$$\gamma_Q^{(1)} \approx \gamma_Q^{(2)} ; U_P^{(1)} - U_P^{(2)} \approx U_Q^{(1)} - U_Q^{(2)}$$

Conversion:

$$N^{(2)} = N^{(1)} + \frac{U_Q^{(1)} - U_Q^{(2)}}{\gamma_Q^{(2)}}$$

Assumption: Constant potential at P

$$\begin{Bmatrix} C_{nm}^{(1)} \\ S_{nm}^{(1)} \end{Bmatrix} = \left(\frac{GM^{(2)}}{GM^{(1)}} \right) \left(\frac{a^{(2)}}{a^{(1)}} \right)^n \begin{Bmatrix} C_{nm}^{(2)} \\ S_{nm}^{(2)} \end{Bmatrix}$$

Disturbing Potential T_P

$$\begin{Bmatrix} \Delta C_{nm}^{(1)} \\ \Delta S_{nm}^{(1)} \end{Bmatrix} = \begin{Bmatrix} C_{nm}^{(1)} \\ S_{nm}^{(1)} \end{Bmatrix} - \begin{Bmatrix} C_{nm}^{*(1)} \\ S_{nm}^{*(1)} \end{Bmatrix}$$

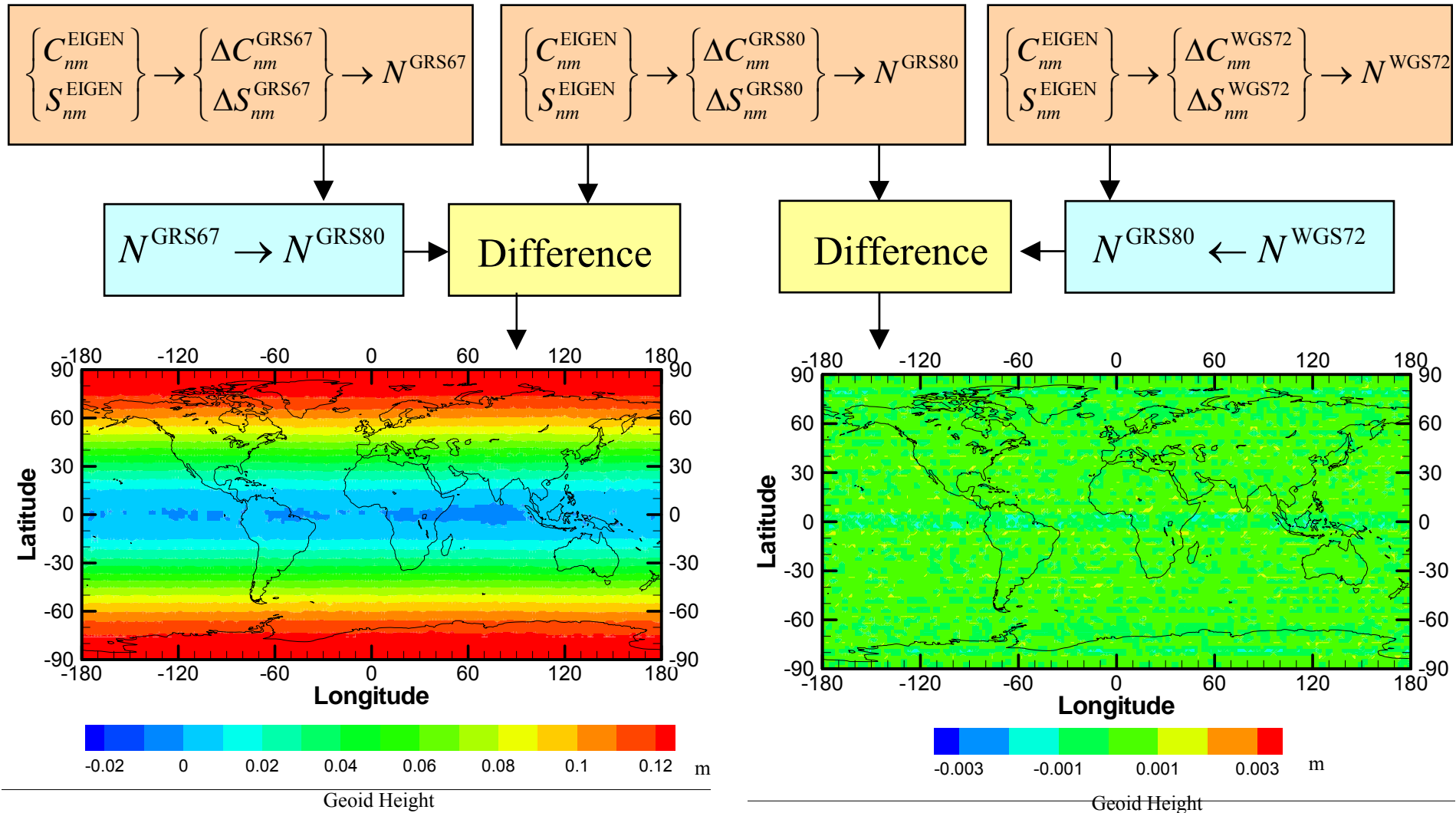
Assumption: $W_P = U_Q$

$$N^{(1)} = \frac{GM^{(1)}}{r\gamma_Q^{(1)}} \sum_{n=0}^N \left(\frac{a^{(1)}}{r} \right)^n \sum_{m=0}^n \left(\Delta C_{nm}^{(1)} R_{nm} + \Delta S_{nm}^{(1)} T_{nm} \right)$$

GOCE & Surface Data

Reference Ellipsoid Conversion Geoid vs. Spherical Harmonic Series Conversion

Closed-Loop Test



GOCE & Applications

Problem: For applications in geodesy, oceanography, solid Earth sciences and sea level research spherical harmonic series of GOCE gravity field model has to be converted to derived gravity field quantities like **geoid heights/height anomalies** and **gravity anomalies**. How to do this conversion correctly according to the needs of the application ?



GOCE High Level Processing Facility

GOCE Level 2 Data Handbook

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Prepared by: The European GOCE Gravity Consortium
EGG-C

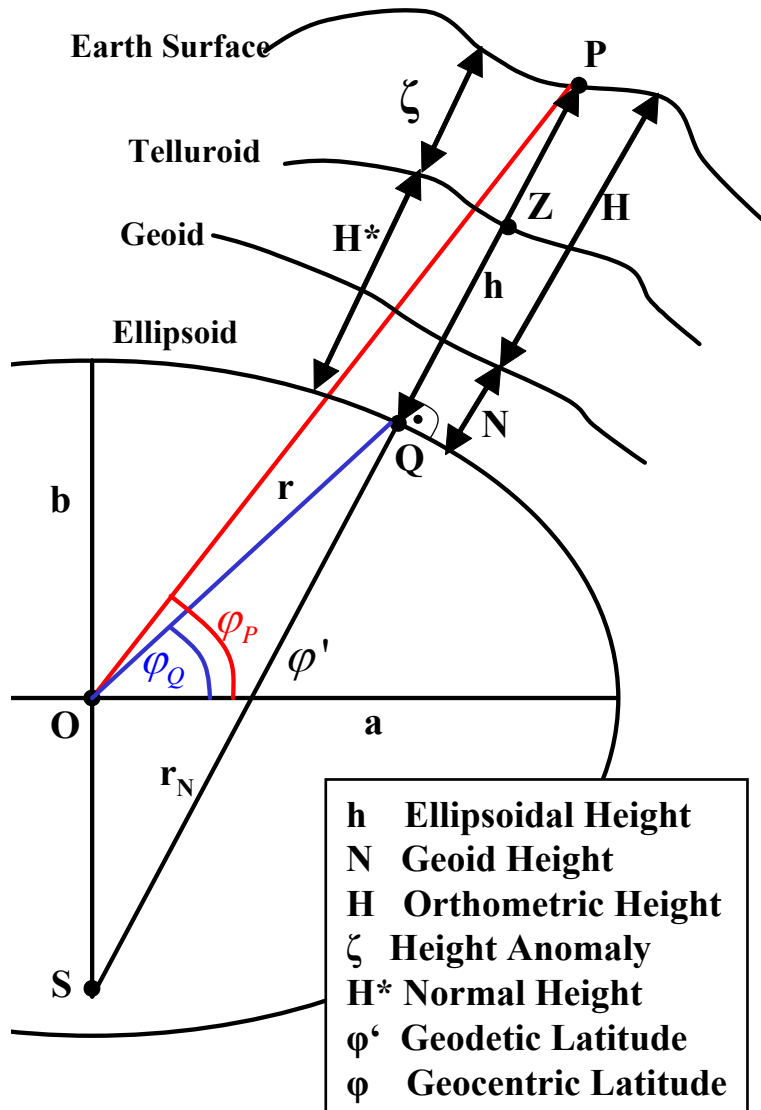
GOCE Level 2 Data Handbook prepared:
(as ESA document)

GOCE Mission

- GOCE Data Processing Overview
- Reference Frames and Time Systems
- Mathematical Conventions
 - Quaternions,
 - Spherical Harmonic Series,
 - Error Propagation
- Geophysical Conventions
 - Approximations for derived quantities
 - Accurate Formulations
- Level 2 Product Definitions
 - Orbits
 - Gravity Fields & Covariance Matrix

GOCE & Applications

Geoid Height vs. Height Anomaly



$$N = \frac{T}{\gamma_Q} = \frac{GM}{r\gamma_Q} \sum_{n=0}^N \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\Delta C_{nm} R_{nm} + \Delta S_{nm} T_{nm})$$

R_{nm}, T_{nm} with ϕ_Q ; $r = r_Q$

$$\zeta = \frac{T}{\gamma_Z} = \frac{GM}{r\gamma_Z} \sum_{n=0}^N \left(\frac{a}{r}\right)^n \sum_{m=0}^n (\Delta C_{nm} R_{nm} + \Delta S_{nm} T_{nm})$$

R_{nm}, T_{nm} with ϕ_P ; $r = r_P$

Effect of height error h on ζ (at 45° latitude)

Height Error [m]	Change in ζ [mm]
1	0.03
10	0.26
100	2,57

Conclusions

In order reach the prospected 1 cm geoid accuracy for GOCE, apart from the observations, the mathematical and geophysical models applied are needed with the same accuracy level. This is specifically important for:

- System transformations and processing standards,
- Corrections and filters needed for combination and validation with surface data, and
- Geophysical conventions applied for derived gravity field quantities in view of their application.

Within the GOCE-GRAND II project these issues are addressed by inter-institutional working groups in close connection with standards and conventions applied in the ESA ground processing.