Alternative GOCE Gradiometer Processing – Wiener Method for Angular Rate Determination

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I. GOCE gradiometer processing steps

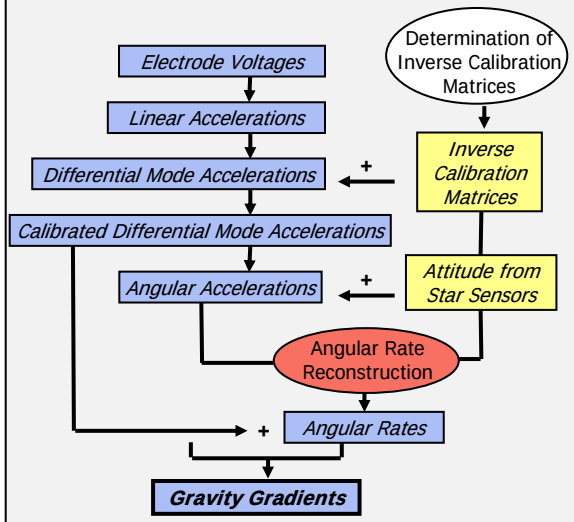

Fig. 1: Overview of the processing steps from the measured electrode voltages to the gravity gradients, which can be used for gravity field determination. In this study an alternative processing method for the step called angular rate reconstruction (ARR, red) has been investigated.

II. Alternative processing strategies

Combination of two star sensors

The two available sets of star sensor (STR) attitude quaternions are combined in such a way, that only the measurements of the very precise boresight axes are used, see also III.

Angular rate determination by Wiener filtering

Within the angular rate reconstruction the attitude information from star sensors and gradiometer are merged. In the nominal processing, this step is done by Kalman filtering in the time domain. In this study, it is done by Wiener filtering in the frequency domain, see also IV.

Computation of inertial attitude quaternions

Within the nominal angular rate reconstruction, besides the inertial angular rates, also the inertial attitude quaternions (IAQ) are determined. For this study the IAQs have been computed by combination of the quaternions from two star sensors (see III) and low-pass filtering them in the frequency domain at 5 mHz.

III. Combination of two star sensors

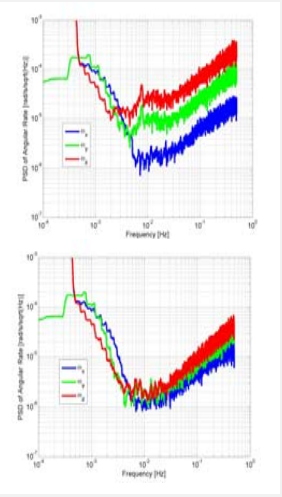

Fig. 2: Power spectral density (PSD) of angular rate (in GRF) from STR 2 (top) and from combination of STR 1 and 2 (bottom)

Each GOCE STR has one ultra sensitive axis, its boresight, whereas the other two directions are less sensitive. When the attitude measurements of the individual STRs are transformed to the gradiometer reference frame (GRF), the less sensitive measurements leak into the very accurate components. This can be avoided by combination of the attitude quaternions from two STRs, using the good axes only.

IV. Angular rate determination by Wiener filtering

The Wiener method for angular rate determination performs a spectral combination of the three angular rate components (of STR and EGG) about the axes of the GRF in the inertial reference frame. The optimal combination is performed by weighting the angular rate components according to their noise PSDs. The weights h_k of the Wiener filter are obtained by:

$$h(STR)_k = \frac{\sigma_k^2(EGG)}{\sigma_k^2(EGG)+\sigma_k^2(STR)}; \quad h(EGG)_k = \frac{\sigma_k^2(STR)}{\sigma_k^2(EGG)+\sigma_k^2(STR)}.$$

(k : frequency)

Based on the stochastic models of STR and EGG (see V) four different sets of STR and EGG noise models have been tested.

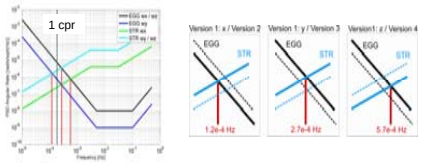

Fig. 3: PSD of assumed STR and EGG angular rate noise – version 1 (left) and versions 2 to 4 (right)

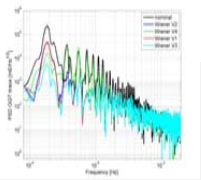

Fig. 4: PSD of GGT trace – impact of different cross-over frequencies between STR and EGG

- **Best results** can be achieved with version 3 of assumed STR and EGG angular rate noise, corresponding to a **cross-over frequency of 0.27 mHz**.
- All further results in this study (see VI and VII) are based on version 3 of assumed noise.

V. Stochastic models

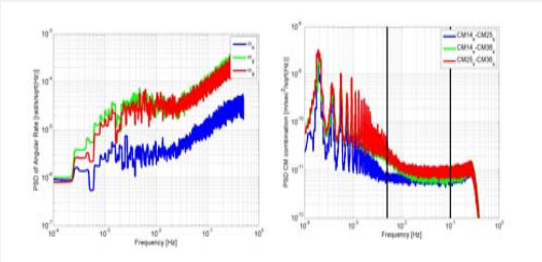

Fig. 5, left: PSD of difference in angular rate (in GRF) of STR 1 and 2

- at high frequencies: f – increase of noise
- at lower frequencies: higher noise than f – increase; peaks of noise at frequencies which are multiples of the 1 cycle per revolution (cpr) frequency

Fig. 5, right: PSD of differences in common mode accelerations in GRF, x-direction

- at high frequencies (in MBW): white noise ~ < 10^{-11} m / s^2 / $\sqrt{\text{Hz}}$
- towards lower frequencies: ~ $1/f^2$ – increase of noise
- peaks of noise at frequencies which are multiples of the 1 cpr frequency

VI. Results – gravity gradients

Four different sets of gravity gradients (GG) have been computed, to analyze the impact of the Wiener method for angular rate determination and the impact of STR combination:

GG set	method	STR combination
A	nominal	no
B	nominal	yes
C	Wiener	no
D	Wiener	yes

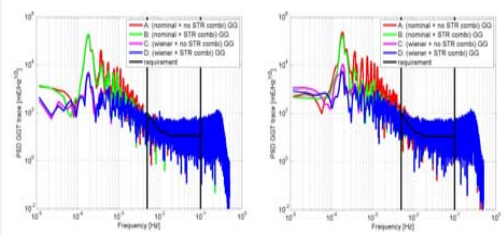

Fig. 6: GGT trace improvement, STR 2 in nominal processing (left) and STR 1 in nominal processing (right)

- **general improvement due to Wiener method below and in the lower MBW**
- **significant reduction of the peaks, which occur as multiples of the 1 cpr frequency**
- some improvement due to STR combination applied to nominal method (especially w.r.t. STR 1)
- no significant change in performance due to STR combination applied to Wiener method

VII. Results – gravity field

To analyze at the level of gravity field modelling the impact of the Wiener method, the STR combination and the new method for computation of the IAQs, four scenarios have been tested, using the Quick-Look Gravity Field Analysis Processor:

scenario	method	STR combination	IAQ
Aa	nominal	no	nominal
Ab	nominal	yes	new
Cb	Wiener	no	new
Db	Wiener	yes	new

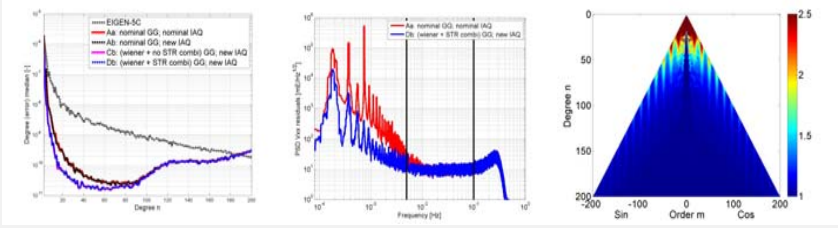

Fig. 7, left: Degree median differences between different scenarios and EIGEN-5C

- **significant improvement due to Wiener method up to degree / order ~ 100**
- no significant change in performance due to STR combination or use of new attitude quaternions

Fig. 7, middle: PSD of the residuals in V_{XX} after gravity field adjustment (difference between adjusted and original observations)

- **general improvement below and in the lower MBW**
- **significant reduction of the peaks, which occur as multiples of the 1 cpr frequency**

Fig. 7, right: Relative improvement in coefficients from solution Aa to Db due to new ARR method (ratio of standard deviations)

- **for some coefficients significant improvement up to degree / order ~ 120**

Reference: Paper submitted to Journal of Geodesy. Alternative method for angular rate determination within the GOCE gradiometer processing; Acknowledgement: We kindly acknowledge the provision of GOCE data via the EOLI-SA portal by ESA.